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Sum rules for the T-odd fragmentation functions

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Abstract

The conservation of the intrinsic transverse momentum during parton fragmentation imposes non-trivial constraints on T-odd fragmentation functions. These significantly enhance the differences between the favoured and unfavoured fragmentation functions, which could be relevant to understand the azimuthal asymmetries of charged pion production observed recently by the HERMES collaboration.

The T-odd effects in partonic fragmentation [1, 2] provide a unique tool for the study of parton polarization. The key point is that they generate a self-analyzing pattern, which connects the parton polarizations in the hard process to angular asymmetries in the produced hadrons.

The fragmentation functions, being the cornerstone of such processes, should be very different from the standard ones. Their characteristic property is that they require the interference of two probability amplitudes with a relative phase, which excludes the usual probabilistic interpretation. This makes the non-perturbative modelling of such effects rather difficult. One possibility is to use an effective quark propagator incorporating the imaginary part [2], another are model calculations in which the phase shift is produced by the Breit-Wigner propagator for a wide hadronic resonances [3, 4]. The available experimental data are also not numerous at the moment. One should note the measurement of longitudinal handedness [4] and the correlation of the Collins fragmentation function [5] in e^+e^--annihilation by the DELPHI collaboration. At the same time, the recent measurements of azimuthal asymmetries in DIS by SMC [6] and HERMES [7] show the need for an extended theoretical treatment. In a complete analysis the effects are represented as convolutions of various distribution and fragmentation functions [8] (and possibly, T-odd *fracture* functions [9]), for which one needs some phenomenological inputs.

In this situation model independent constraints for T-odd fragmentation functions are most welcome. In the present article we present a method to derive such constraints, which is based on the conservation of the intrinsic transverse momentum of hadrons in partons. The resulting zero sum rule for the fragmentation function $H_1^{\perp(1)}$ allows to understand the flavour dependence of the azimuthal asymmetry observed at HERMES.

To be more specific, let us start with the definition of the fragmentation functions [2]

$$\begin{aligned}
&\int \frac{dx^- d^2x_T}{12(2\pi)^3} \exp\left(iP^+\frac{x^-}{z} + iP_T\frac{x_T}{z}\right)\ Tr_{Dirac}\gamma^\mu \sum_{P,X} <0|\psi(0)|P,X><P,X|\bar{\psi}(x)|0> \\
&\qquad = D(z,k_T,\mu^2)P^\mu \\
&\int \frac{dx^- d^2x_T}{12(2\pi)^3} \exp\left(iP^+\frac{x^-}{z} + iP_T\frac{x_T}{z}\right)\ Tr_{Dirac}i\sigma^{\mu\nu} \sum_{P,X} <0|\psi(0)|P,X><P,X|\bar{\psi}(x)|0> \\
&\qquad = H_1^\perp(z,k_T,\mu^2)\frac{P^\mu k_T^\nu}{M}
\end{aligned} \tag{1}$$

Here k_T and M are the intrinsic transverse momentum of the parton and a mass parameter of order of the jet mass. We have also given, for comparison, the expression for the standard spin-averaged fragmentation function D. The hadron and parton momenta are related by $P^+ = zk^+, P_T = -zk_T$

Let us immediately come to the physical interpretation. For D it is most straigtforward. We adopt the normalization conditions of [2] (differing from that of [8] by a factor of z) so that $P(z,k_T) = D(z,k_T)/z$ is the probability density to find a specific hadron with the specified momentum. The transverse momentum integrated probability density is therefore

$$P(z) = \int d^2P_T \frac{D(z, -\frac{P_T}{z})}{z} = z\int d^2k_T D(z,k_T) \equiv zD(z), \tag{2}$$

so that longitudinal momentum conservation takes the form

$$\sum_h \int_0^1 dz z^2 D(z) = 1. \tag{3}$$

Let us pass on to T-odd fragmentation function. Although there is no direct probabilistic interpretation (the relative phase shift between the matrix elements

$$< 0|\sigma^{\mu\nu}\psi(0)|P, X >, < P, X|\bar{\psi}(x)|0 > \tag{4}$$

is actually crucial), it may still be considered as a quark spin dependent term in the differential cross-section, so that the corresponding probability density is proportional to

$$\frac{H_1^\perp(z, k_T)}{z} k_T \cos\phi \tag{5}$$

Here ϕ is the azimuthal angle with respect to the plane of the given component of transverse momentum. One can now immediately write the *transverse* momentum conservation as

$$\sum_h \int dz \int d^2P_T P_T k_T cos^2\phi \frac{H_1^\perp(z, -\frac{P_T}{z})}{z} \sim \sum_h \int dz z^2 \int d^2k_T k_T^2 H_1^\perp(z, k_T) = 0. \tag{6}$$

This equality leads to the conservation of each of the two components of transverse momentum, due to the arbitrary choice of the angle ϕ. The integration over ϕ is factored out. The integrated quantity is then proportional to the transverse momentum averaged function [8]

$$H_1^{\perp(1)}(z) = \int d^2k_T \frac{k_T^2}{2M^2} H_1^\perp(z, k_T). \tag{7}$$

It then follows that

$$\sum_h \int dz z^2 H_1^{\perp(1)}(z) = 0, \tag{8}$$

which is our main result.

Several additional comments might help to better understand this formula.

First, note that the k_T oddness required by the T-odd nature was absolutely crucial in obtaining the result. While the conservation of the transverse momentum components applies to all k_T-dependent functions, it gives only non-trivial constraints for *odd* powers of k_T. The similar contribution to $D(z, k_T)$ is zero just because it is an even function of k_T. Therefore, similar sum rules hold for all k_T-odd fragmentation functions. Moreover, similar sum rules can also be derived for functions which do not requiring intrinsic k_T at all, like for the twist-3 T-odd fragmentation function c_V [10], which describes the fragmentation of unpolarized quarks into polarized baryons.

Second, the chirality of the function is actually inessential in the presented derivation, as the latter does not make direct reference to the energy-momentum operator, which is chiral even. For the actual function $H_1^{\perp(1)}$ one should think about momentum conservation for processes initiated by the tensor quark currents, rather than about matrix elements of the momentum operators.

The immediate consequence of our sum rule is a larger difference between favored (for $z \sim 1$) and unfavored fragmentation. If the favored one is positive for large z, there must be compensating negative contribution at lower z. For the unfavored fragmentation function which probes generally smaller z values the resulting cancellations should then be more pronounced. Therefore we expect that for all T-odd fragmentation functions the contributions from non-leading parton fragmentation will be severely suppressed compared to the normal fragmentation function $D(z)$. The resulting asymmetries should be much smaller for non-leading hadrons than for leading ones. This behaviour actually shows up in a comparison of the double semi-inclusive π^+ and π^- asymmetries as measured by HERMES [11] with the π^+ and π^- azimuthal single spin asymmetries, measured by the same collaboration [7, 12].

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